The Dividend Discount Model
Proof: Expected Total Return Equals The Discount Rate

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In this white paper we will prove the following statement...

Statement: If you discount cash flow at the cost of capital then your expected future total return will be the cost of capital.

Company Value

We will define the variable R_t to be annualized revenue at time t and the variable μ to be the continuous-time revenue growth rate. The equation for annualized revenue at time t is...

$$R_t = R_0 \,\mathrm{Exp}\left\{\mu\, t\right\} \tag{1}$$

We will define the variable A_t to be total tangible assets at time t and the variable θ to be the ratio of tangible assets to annualized revenue. Using Equation (1) above, the equation for tangible assets at time t is...

$$A_t = \theta R_t = \theta R_0 \,\mathrm{Exp}\left\{\mu\, t\right\} \tag{2}$$

The equation for the derivative of Equation (2) above with respect to time is...

$$\frac{\delta A_t}{\delta t} = \mu\,\theta R_0 \,\mathrm{Exp}\left\{\mu\, t\right\} \text{ ...such that... } \delta A_t = \mu\,\theta R_0 \,\mathrm{Exp}\left\{\mu\, t\right\}\delta t \tag{3}$$

We will define the variable N_t to be annualized net income at time t and the variable π to be the after-tax return on tangible assets. Using Equation (2) above, the equation for annualized net income is...

$$N_t = \pi A_t = \pi\,\theta R_0 \,\mathrm{Exp}\left\{\mu\, t\right\} \tag{4}$$

We will define the variable C_t to be annualized net cash flow at time t. The generic equation for annualized net cash flow is...

$$C_t = \text{Annualized net income (i.e. profit/(loss))} - \text{Annualized change in assets (i.e. investment)} \tag{5}$$

Using the Equations (3) and (4) above, we can rewrite Equation (5) above as...

$$C_t = N_t - \delta A_t = \theta\left(\pi - \mu\right) R_0 \,\mathrm{Exp}\left\{\mu\, t\right\} \tag{6}$$

We will define the variable V_t to be company value at time t and the variable κ to be the continuous-time risk-adjusted discount rate. Using Equation (6) above, the equation for company value at time t from the perspective of time zero is...

$$V_t = \int_t^{\infty} C_s \,\mathrm{Exp}\left\{-\kappa\,(s-t)\right\}\delta s \tag{7}$$

Using Appendix Equations (25) and (26) below, the solution to Equation (7) above is...

$$V_t = \frac{\pi - \mu}{\kappa - \mu}\,\theta R_0 \,\text{Exp}\left\{\mu\, t\right\} \tag{8}$$

Note that the derivative of Equation (8) above with respect to time is...

$$\frac{\delta V_t}{\delta t} = \mu\, V_t \ \text{...such that...}\ \delta V_t = \mu\,\frac{\pi - \mu}{\kappa - \mu}\,\theta R_0 \,\text{Exp}\left\{\mu\, t\right\}\delta t \tag{9}$$

Share Price

We will define the variable S_t share price at time t and the variable N to be the number of common shares outstanding. Using Equation (8) above, the equation for share price at time t is...

$$S_t = V_t\, N^{-1} = \frac{\pi - \mu}{\kappa - \mu}\,\theta R_0 \,\text{Exp}\left\{\mu\, t\right\} N^{-1} \tag{10}$$

Using Equations (9) and (10) above, the equation for shareholder capital gains realized over the infinitesimally small time interval $[t, t + \delta t]$ is...

$$\text{Capital gains} = \delta V_t\, N^{-1} = \mu\,\frac{\pi - \mu}{\kappa - \mu}\,\theta R_0 \,\text{Exp}\left\{\mu\, t\right\} N^{-1}\,\delta t = \mu\, S_t\,\delta t \tag{11}$$

Using Equation (11) above, the equation for cumulative capital gains realized over the time interval $[t, t + \Delta]$ is...

$$\text{Cumulative capital gains} = \int\limits_t^{t+\Delta} \mu\,\frac{\pi - \mu}{\kappa - \mu}\,\theta R_0 \,\text{Exp}\left\{\mu\, s\right\} N^{-1}\,\delta s \tag{12}$$

Using Appendix Equation (27) below, the solution to Equation (12) above is...

$$\text{Cumulative capital gains} = \frac{\pi - \mu}{\kappa - \mu}\,\theta R_0\, N^{-1}\left[\text{Exp}\left\{\mu\,(t + \Delta)\right\} - \text{Exp}\left\{\mu\, t\right\}\right] \tag{13}$$

We will define the variable ϕ to be the dividend yield. If each share earns $\kappa\, S_t\,\delta t$ over the infinitesimally small time interval $[t, t + \delta t]$ and per Equation (11) above capital gains are $\mu\, S_t\,\delta t$, the equation for the dividend yield is...

$$\phi = \kappa - \mu \tag{14}$$

Using Equations (8), (10) and (14) above, the equation for dividends received by the shareholder over the infinitesimally small time interval $[t, t + \delta t]$ is...

$$\text{Dividend income} = \phi\, V_t\, N^{-1}\,\delta t = \phi\,\frac{\pi - \mu}{\kappa - \mu}\,\theta R_0 \,\text{Exp}\left\{\mu\, t\right\} N^{-1}\,\delta t = \phi\, S_t\,\delta t \tag{15}$$

Using Equation (15) above, the equation for cumulative dividends received over the time interval $[t, t + \Delta]$ is...

$$\text{Cumulative dividend income} = \int\limits_t^{t+\Delta} \phi\,\frac{\pi - \mu}{\kappa - \mu}\,\theta R_0 \,\text{Exp}\left\{\mu\, s\right\} N^{-1}\,\delta s \tag{16}$$

Using Appendix Equation (28) below, the solution to Equation (16) above is...

$$\text{Cumulative dividend income} = \frac{\phi}{\mu}\,\frac{\pi - \mu}{\kappa - \mu}\,\theta R_0\, N^{-1}\left[\text{Exp}\left\{\mu\,(t + \Delta)\right\} - \text{Exp}\left\{\mu\, t\right\}\right] \tag{17}$$

Proof

We want to prove the following statement...

$$\text{Total return} = \int\limits_t^{t+\Delta} \kappa \, S_t \, \delta t \tag{18}$$

Using Equation (10) above, we can rewrite Equation (18) above as..

$$\text{Cumulative total return} = \int\limits_t^{t+\Delta} \kappa \, \frac{\pi - \mu}{\kappa - \mu} \, \theta R_0 \, \text{Exp}\left\{\mu \, t\right\} N^{-1} \, \delta t \tag{19}$$

Using Appendix Equation (29) below, the solution to Equation (19) above is...

$$\text{Cumulative total return} = \frac{\kappa}{\mu} \, \frac{\pi - \mu}{\kappa - \mu} \, \theta R_0 \, N^{-1} \left[\text{Exp}\left\{\mu \, (t + \Delta)\right\} - \text{Exp}\left\{\mu \, t\right\}\right] \tag{20}$$

Note that we can rewrite Equation (14) above as...

$$\text{if... } \phi = \kappa - \mu \text{ ...then... } \kappa = \phi + \mu \tag{21}$$

Using Equation (21) above, we can rewrite Equation (20) as...

$$\begin{aligned} \text{Cumulative total return} &= \frac{\phi + \mu}{\mu} \, \frac{\pi - \mu}{\kappa - \mu} \, \theta R_0 \, N^{-1} \left[\text{Exp}\left\{\mu \, (t + \Delta)\right\} - \text{Exp}\left\{\mu \, t\right\}\right] \\ &= \left(\frac{\phi}{\mu} + 1\right) \frac{\pi - \mu}{\kappa - \mu} \, \theta R_0 \, N^{-1} \left[\text{Exp}\left\{\mu \, (t + \Delta)\right\} - \text{Exp}\left\{\mu \, t\right\}\right] \end{aligned} \tag{22}$$

Note that Equation (22) (cumulative total return) equals Equation (13) (cumulative capital gains) plus Equation (17) (cumulative dividend income). **Our global statement above is proved**.

Appendix

A. The solution to the following integral is...

$$\begin{aligned} V_t &= \int\limits_t^{\infty} C_s \, \text{Exp}\left\{-\kappa \, (s - t)\right\} \delta s \\ &= \int\limits_t^{\infty} \theta \left(\pi - \mu\right) R_0 \, \text{Exp}\left\{\mu \, s\right\} \text{Exp}\left\{-\kappa \, (s - t)\right\} \delta s \\ &= \theta \left(\pi - \mu\right) R_0 \, \text{Exp}\left\{\kappa \, t\right\} \int\limits_t^{\infty} \text{Exp}\left\{(\mu - \kappa) \, s\right\} \delta s \\ &= \frac{\theta \, (\pi - \mu)}{\mu - \kappa} \, R_0 \, \text{Exp}\left\{\kappa \, t\right\} \left[\text{Exp}\left\{(\mu - \kappa) \, \infty\right\} - \text{Exp}\left\{(\mu - \kappa) \, t\right\}\right] \end{aligned} \tag{23}$$

Given the binding constraint revenue growth rate < the discount rate (i.e. $\mu < \kappa$)...

$$\text{if... } \mu < \kappa \text{ ...then... } \lim_{s \to \infty} \text{Exp}\left\{(\mu - \kappa) \, s\right\} = 0 \tag{24}$$

Using Equation (24) above, we can rewrite Equation (23) above as...

$$\begin{aligned} V_t &= \frac{\theta \, (\pi - \mu)}{\mu - \kappa} \, R_0 \, \text{Exp}\left\{\kappa \, t\right\} \left[0 - \text{Exp}\left\{(\mu - \kappa) \, t\right\}\right] \\ &= \frac{\theta \, (\pi - \mu)}{\kappa - \mu} \, R_0 \, \text{Exp}\left\{\kappa \, t\right\} \text{Exp}\left\{(\mu - \kappa) \, t\right\} \\ &= \frac{\theta \, (\pi - \mu)}{\kappa - \mu} \, R_0 \, \text{Exp}\left\{\mu \, t\right\} \end{aligned} \tag{25}$$

B. The deriviative of Equation (25) above with respect to time is...

$$\frac{\delta V_t}{\delta t} = \mu \, \frac{\theta \, (\pi - \mu)}{\kappa - \mu} \, R_0 \, \text{Exp} \left\{ \mu \, t \right\} = \mu \, V_t \;\; \text{...such that...} \;\; \delta V_t = \mu \, V_t \, \delta t \tag{26}$$

C. The solution to the following integral is...

$$\begin{aligned} I &= \int\limits_t^{t+\Delta} \mu \, \frac{\pi - \mu}{\kappa - \mu} \, \theta R_0 \, \text{Exp} \left\{ \mu \, s \right\} N^{-1} \, \delta s \\ &= \mu \, \frac{\pi - \mu}{\kappa - \mu} \, \theta R_0 \, N^{-1} \int\limits_t^{t+\Delta} \text{Exp} \left\{ \mu \, s \right\} \delta s \\ &= \mu^{-1} \, \mu \, \frac{\pi - \mu}{\kappa - \mu} \, \theta R_0 \, N^{-1} \left[\text{Exp} \left\{ \mu \, t + \Delta \right\} - \text{Exp} \left\{ \mu \, t \right\} \right] \\ &= \frac{\pi - \mu}{\kappa - \mu} \, \theta R_0 \, N^{-1} \left[\text{Exp} \left\{ \mu \, (t + \Delta) \right\} - \text{Exp} \left\{ \mu \, t \right\} \right] \end{aligned} \tag{27}$$

D. The solution to the following integral is...

$$\begin{aligned} I &= \int\limits_t^{t+\Delta} \phi \, \frac{\pi - \mu}{\kappa - \mu} \, \theta R_0 \, \text{Exp} \left\{ \mu \, s \right\} N^{-1} \, \delta s \\ &= \phi \, \frac{\pi - \mu}{\kappa - \mu} \, \theta R_0 \, N^{-1} \int\limits_t^{t+\Delta} \text{Exp} \left\{ \mu \, s \right\} \delta s \\ &= \mu^{-1} \, \phi \, \frac{\pi - \mu}{\kappa - \mu} \, \theta R_0 \, N^{-1} \left[\text{Exp} \left\{ \mu \, t + \Delta \right\} - \text{Exp} \left\{ \mu \, t \right\} \right] \\ &= \frac{\phi}{\mu} \, \frac{\pi - \mu}{\kappa - \mu} \, \theta R_0 \, N^{-1} \left[\text{Exp} \left\{ \mu \, (t + \Delta) \right\} - \text{Exp} \left\{ \mu \, t \right\} \right] \end{aligned} \tag{28}$$

E. The solution to the following integral is...

$$\begin{aligned} I &= \int\limits_t^{t+\Delta} \kappa \, \frac{\pi - \mu}{\kappa - \mu} \, \theta R_0 \, \text{Exp} \left\{ \mu \, s \right\} N^{-1} \, \delta s \\ &= \kappa \, \frac{\pi - \mu}{\kappa - \mu} \, \theta R_0 \, N^{-1} \int\limits_t^{t+\Delta} \text{Exp} \left\{ \mu \, s \right\} \delta s \\ &= \mu^{-1} \, \kappa \, \frac{\pi - \mu}{\kappa - \mu} \, \theta R_0 \, N^{-1} \left[\text{Exp} \left\{ \mu \, t + \Delta \right\} - \text{Exp} \left\{ \mu \, t \right\} \right] \\ &= \frac{\kappa}{\mu} \, \frac{\pi - \mu}{\kappa - \mu} \, \theta R_0 \, N^{-1} \left[\text{Exp} \left\{ \mu \, (t + \Delta) \right\} - \text{Exp} \left\{ \mu \, t \right\} \right] \end{aligned} \tag{29}$$